Forum for Electromagnetic Research Methods and Application Technologies (FERMAT)

Non-reciprocal devices based on magnetostatically biased graphene: state of art and theoretical limits

Michele Tamagnone[1], Arya Fallahi[2], Juan R. Mosig[1], Julien Perruisseau-Carrier[3]

[1] Laboratory of Electromagnetics and Antennas (LEMA), EPFL, Lausanne, Switzerland
[2] DESY-Center for Free-Electron Laser Science (CFEL), Notkestrasse 85, D-22607 Hamburg, Germany
[3] Deceased. Was with the Adaptive MicroNano Wave Systems, EPFL, Lausanne, Switzerland

Abstract:

The potential of graphene for use in non-reciprocal terahertz applications was evidenced by recent demonstrations of the Faraday rotators and microwave isolators. These promising yet preliminary results raise crucial questions: what is the optimal performance achievable by more complex designs using multilayer structures, graphene patterning, metal additions, or a combination of these approaches, and can this optimum design be achieved in practice? We provide an answer to these questions by showing the existence of an upper bound on the performances of passive graphene non reciprocal devices, and we show optimal designs able to reach these performances. The bound depends uniquely on the conductivity of graphene under magnetic bias, and hence enables an estimation of the best possible performances prior to any design. We believe that these results constitute an important roadmap for future devices based on graphene's non-reciprocity.

Keywords: graphene, isolator, gyrator, non-reciprocity, terahertz.

References:

[1] M. Tamagnone, A. Fallahi, J. R. Mosig, and J. Perruisseau-Carrier, "Fundamental limits and near-optimal design of graphene modulators and non-reciprocal devices," *Nat Photon*, vol. **8**, pp. 556-563, 2014.
[2] M. Tamagnone, A. Fallahi, J. R. Mosig and J. Perruisseau-Carrier. Theoretical Limits of Graphene Terahertz Non-Reciprocal Devices. 9th European Conference on Antennas and Propagation (EuCAP), Lisbon, Portugal, April, 12-17, 2015

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Outline

- Introduction on non-reciprocal devices
- THz isolators, state of the art
- Graphene non-reciprocity devices: state of the art
- Graphene non-reciprocity theoretical upper bound
- Conclusions

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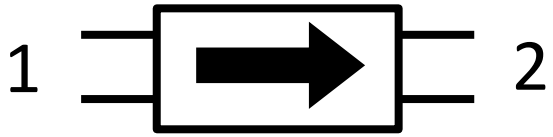
Non reciprocal devices

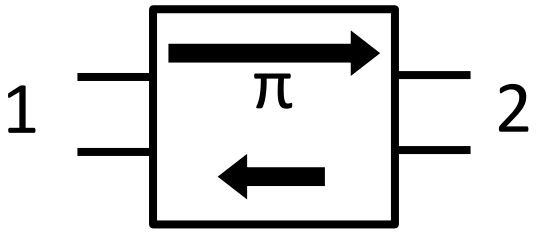
- Isolator (2 ports)

$$S = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$



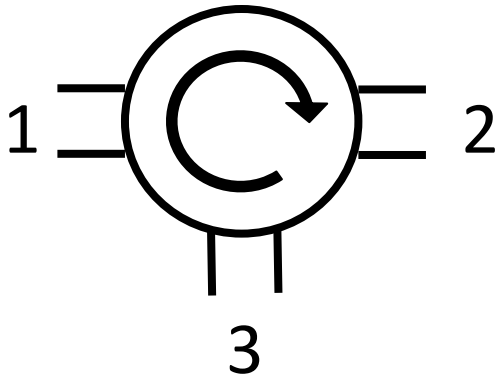
- Gyrator (2 ports)

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

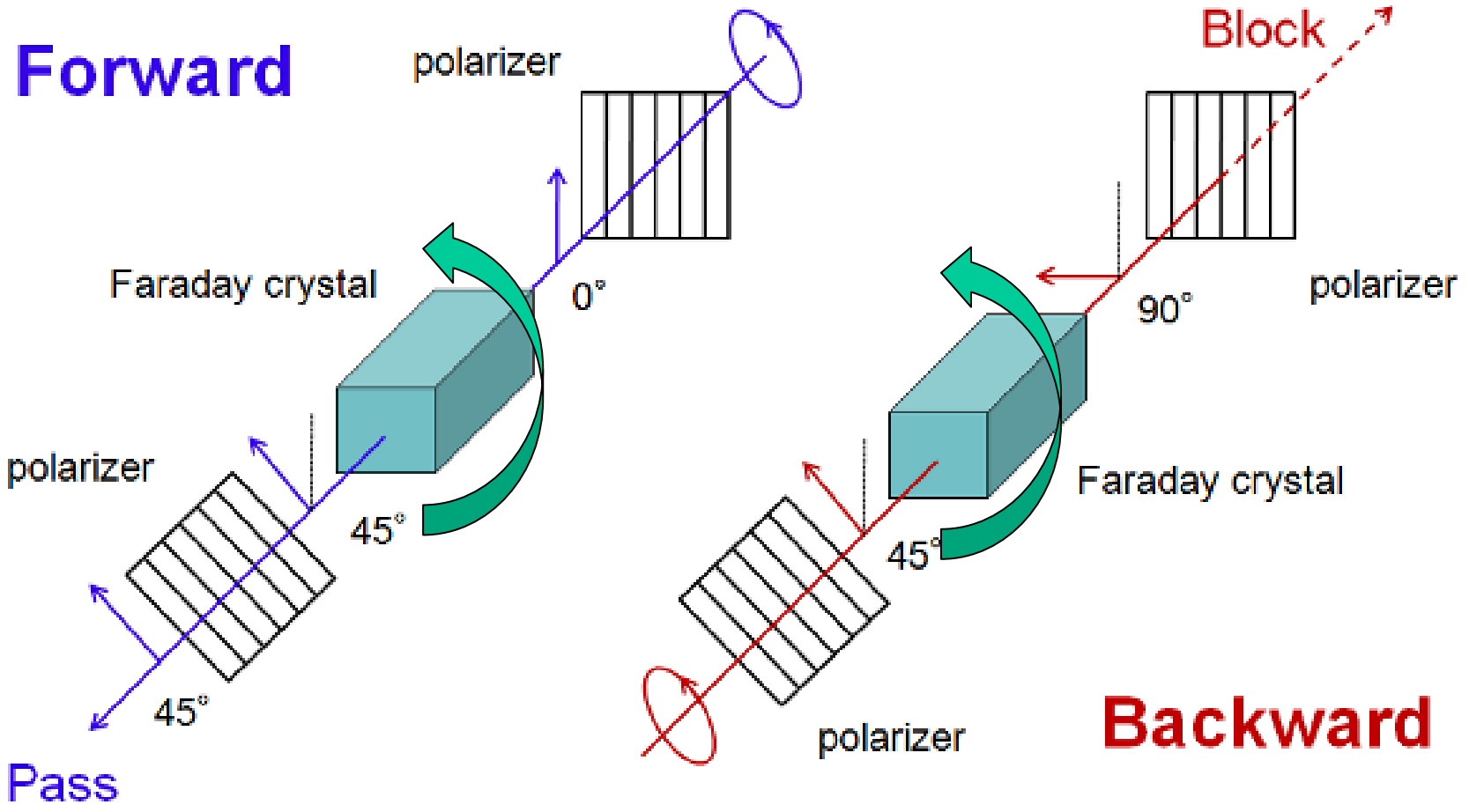


- Circulator (3+ ports)

$$S = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$



Faraday isolator



- T. Amemiya1 and Y. Nakano, “Single Mode Operation of 1.5-μm Waveguide Optical Isolators Based on the Nonreciprocal-loss Phenomenon”, Intech

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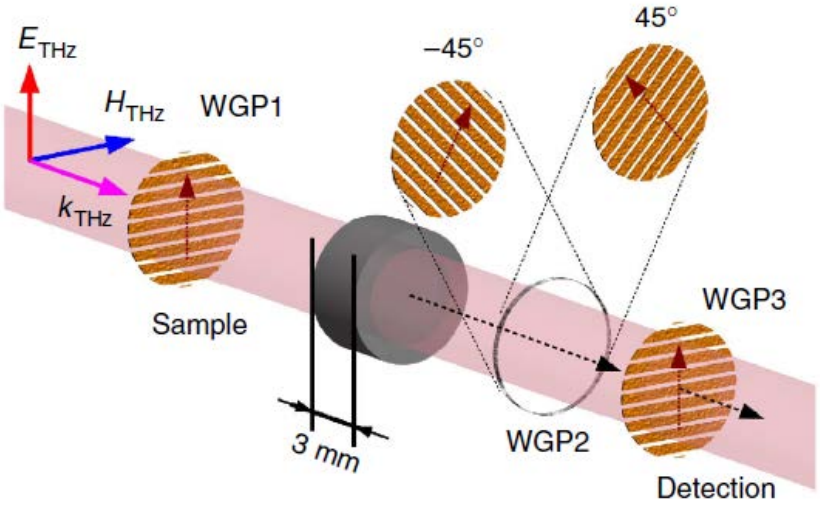
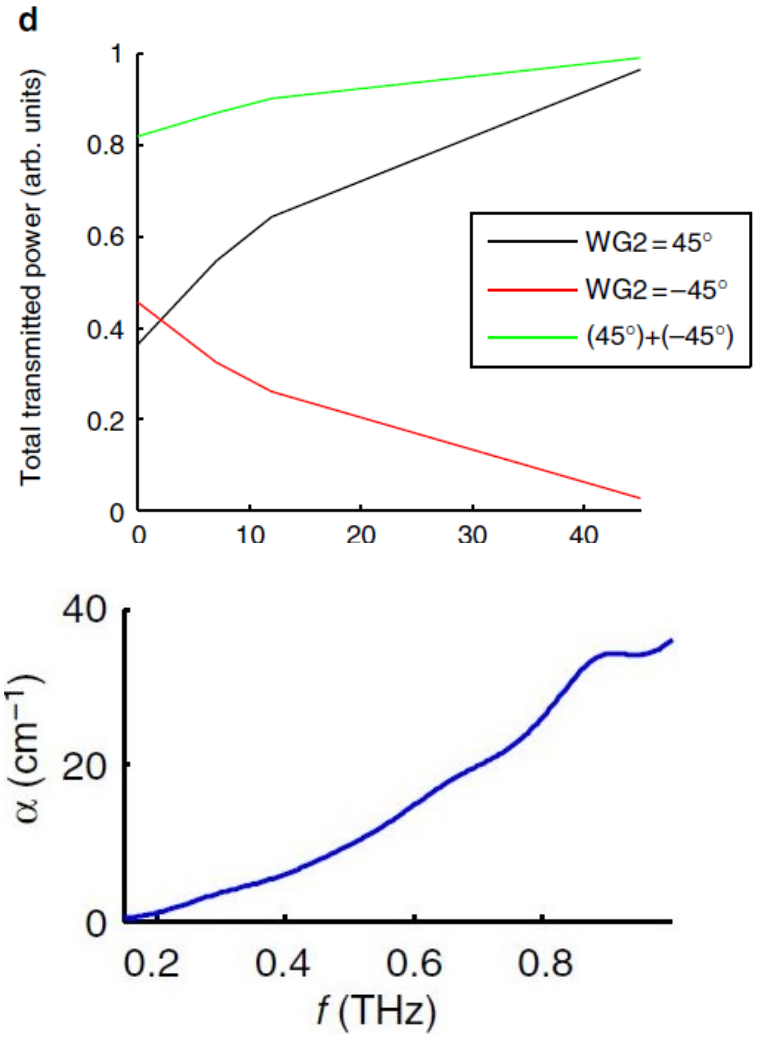
THz Isolators: state of art

- THz isolator: open challenge
- First attempt: $SrFe_{12}O_{19}$
  - Good isolation, but
  - Max 0.8 THz
  - 25 dB of loss at 800GHz





➢ Shalaby, M., Peccianti, M., Ozturk, Y. & Morandotti, R. A magnetic non-reciprocal isolator for broadband terahertz operation. Nat Commun 4, 1558 (2013)

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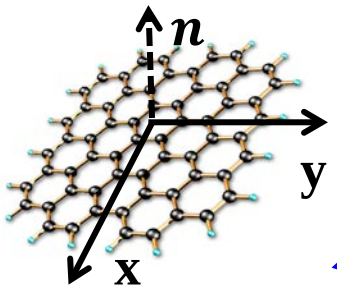
Graphene conductivity

- Graphene is 2D → entirely described by a surface conductivity tensor:



$$\boldsymbol{J} = \boldsymbol{\sigma E} \qquad \boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} = \begin{pmatrix} \sigma_d & \sigma_o \\ -\sigma_o & \sigma_d \end{pmatrix}$$

$$\sigma_d(\omega, T, \tau, \mu_c(E_{bias}), B_{bias})$$

$$\sigma_o(\omega, T, \tau, \mu_c(E_{bias}), B_{bias})$$

$$\sigma_{\mathrm{D}} = \sigma_{\mathrm{DR}} - j\sigma_{\mathrm{DJ}} = j\frac{q_e^2 v_{\mathrm{F}}^2 \left| q_e B_{\mathrm{bias}} \right| (\omega - j2\Gamma)\hbar}{\pi} \cdot \sum_{n=0}^{\infty} \left\{ \frac{n_{\mathrm{F}}(M_n) - n_{\mathrm{F}}(M_{n+1}) + n_{\mathrm{F}}(-M_{n+1}) - n_{\mathrm{F}}(-M_n)}{(M_{n+1} - M_n)^2 - (\omega - j2\Gamma)^2 \hbar^2} \cdot \left(1 - \frac{\Delta^2}{M_n M_{n+1}}\right) \frac{1}{M_{n+1} - M_n} + \frac{n_{\mathrm{F}}(-M_n) - n_{\mathrm{F}}(M_{n+1}) + n_{\mathrm{F}}(-M_{n+1}) - n_{\mathrm{F}}(M_n)}{(M_{n+1} + M_n)^2 - (\omega - j2\Gamma)^2 \hbar^2} \cdot \left(1 + \frac{\Delta^2}{M_n M_{n+1}}\right) \frac{1}{M_{n+1} + M_n} \right\}$$

$$\sigma_{\mathrm{O}} = \sigma_{\mathrm{OR}} - j\sigma_{\mathrm{OJ}} = -\frac{q_e^3 v_{\mathrm{F}}^2 B_{\mathrm{bias}}}{\pi} \cdot \sum_{n=0}^{\infty} \left[n_{\mathrm{F}}(M_n) - n_{\mathrm{F}}(M_{n+1}) + n_{\mathrm{F}}(-M_{n+1}) - n_{\mathrm{F}}(-M_n) \right] \cdot \left\{ \left(1 - \frac{\Delta^2}{M_n M_{n+1}}\right) \frac{1}{(M_{n+1} - M_n)^2 - (\omega - j2\Gamma)^2 \hbar^2} + \left(1 + \frac{\Delta^2}{M_n M_{n+1}}\right) \frac{1}{(M_{n+1} + M_n)^2 - (\omega - j2\Gamma)^2 \hbar^2} \right\}$$

$$M_n = \sqrt{\Delta^2 + 2nv_{\mathrm{F}}^2 \left| q_e B_{\mathrm{bias}} \right| \hbar}.$$

Maxwell boundary condition: $\boldsymbol{n} \times (\boldsymbol{H_1} - \boldsymbol{H_2}) = \boldsymbol{J} = \boldsymbol{\sigma E}$

➢ V. P. Gusynin, S. G. Sharapov J. B. Carbotte, "On the universal AC optical background in graphene", New. J. Physics, 2009.

Graphene conductivity

- The Drude-Lorentz model approximation
  - At room temperature and for frequencies up to few THz, for $\mu_c \gg 2\hbar\omega_c$, the interband transitions can be neglected

    → Drude model good approximation

$$\sigma_d(\omega, B_0) = \sigma_0 \frac{1 + j\omega\tau}{(\omega_c\tau)^2 + (1 + j\omega\tau)^2}$$

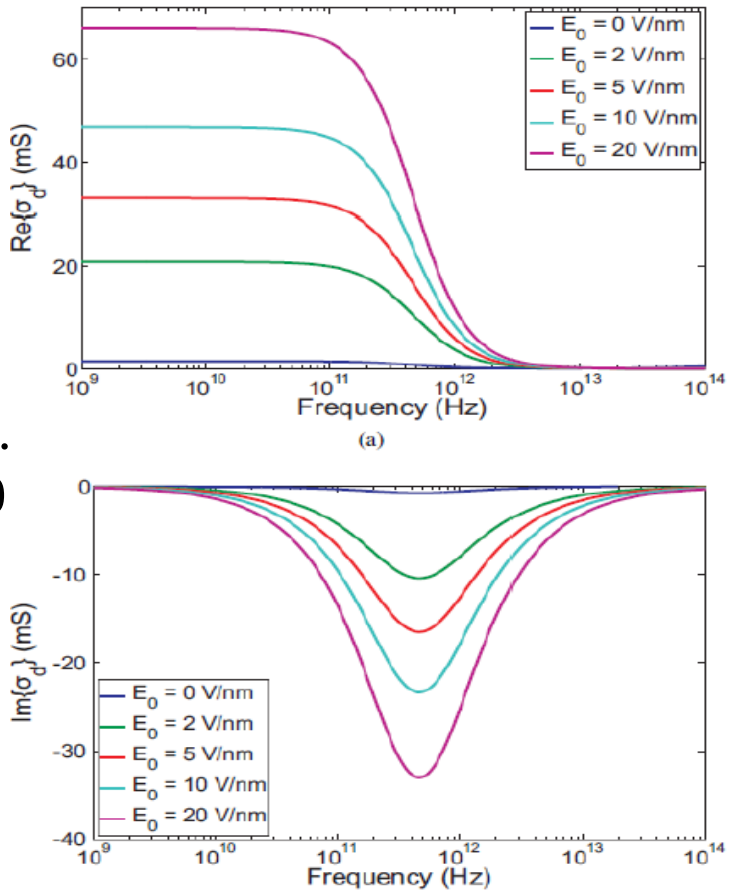
$$\sigma_o(\omega, B_0) = \sigma_0 \frac{\omega_c\tau}{(\omega_c\tau)^2 + (1 + j\omega\tau)^2}$$

$$\sigma_0 = \frac{2e^2\tau}{\pi\hbar^2} k_B T \ln\left(2\cosh\frac{\mu_c}{2k_BT}\right)$$

DC cond. with B=0

$$\omega_c = \frac{eB_0 v_F^2}{\mu_c}$$

Cyclotron frequency

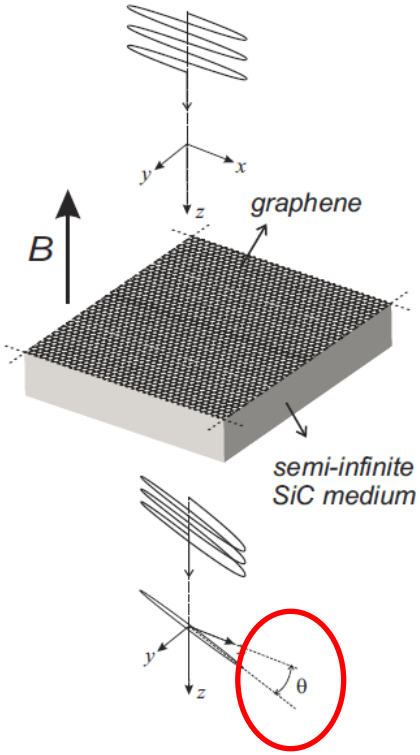


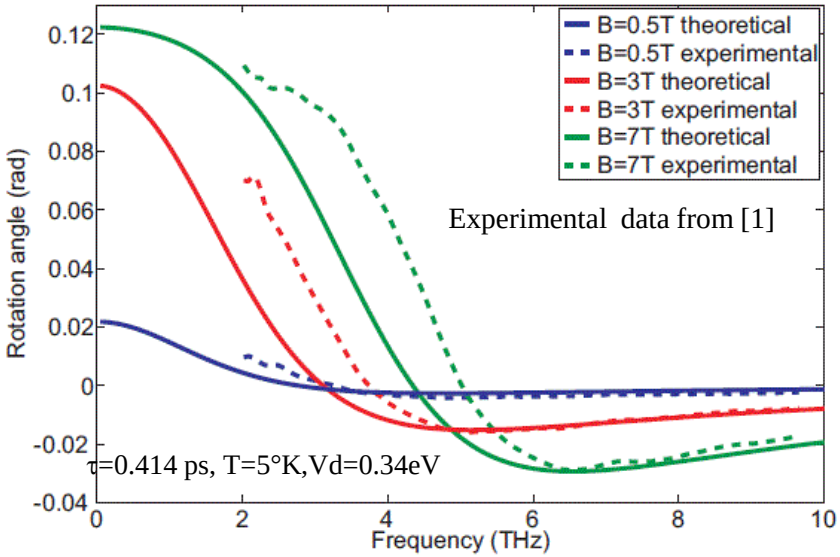
Faraday roatation

- Faraday rotation in uniform graphene layers
  - B-bias → non-diagonal conductivity → Faraday rotation [1]

$$\bar{\bar{\sigma}} = \begin{pmatrix} \sigma_d & \sigma_o \\ -\sigma_o & \sigma_d \end{pmatrix}$$

'Hall conductivity'





I. Crassee et al. "Giant Faraday rotation in single and multilayer graphene," Nature Phys., 2010.

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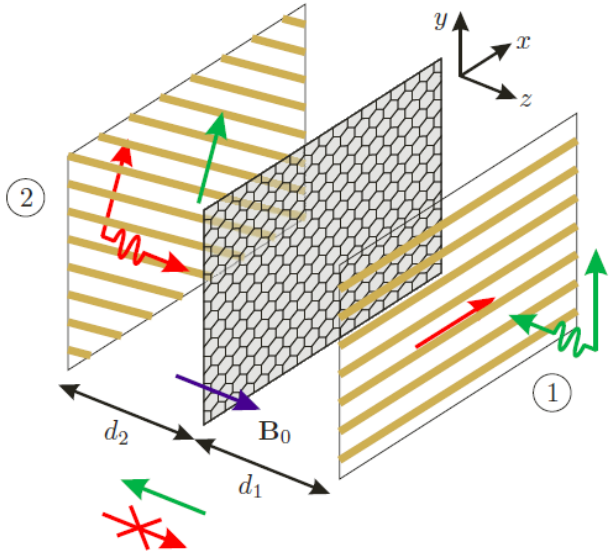
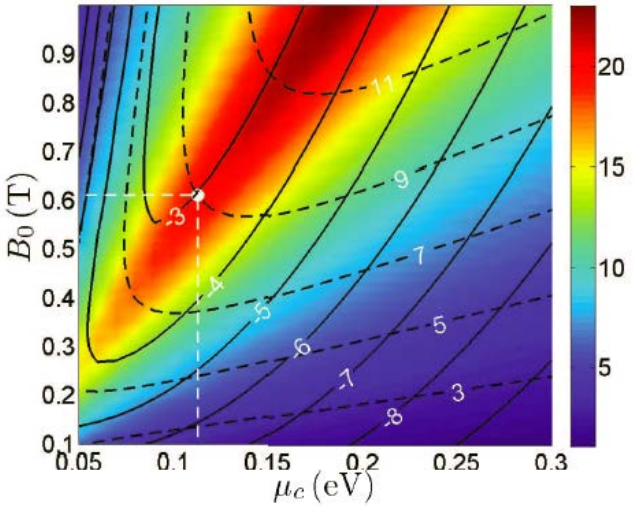
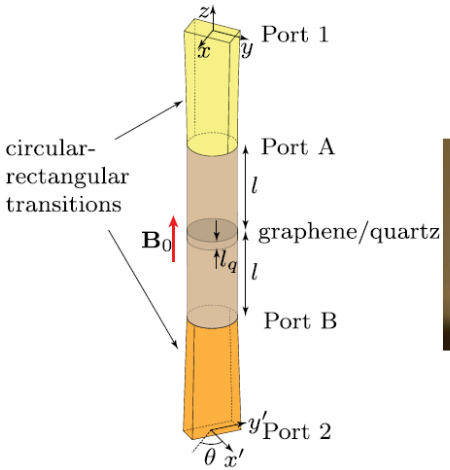
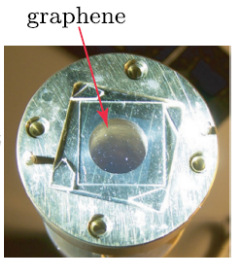
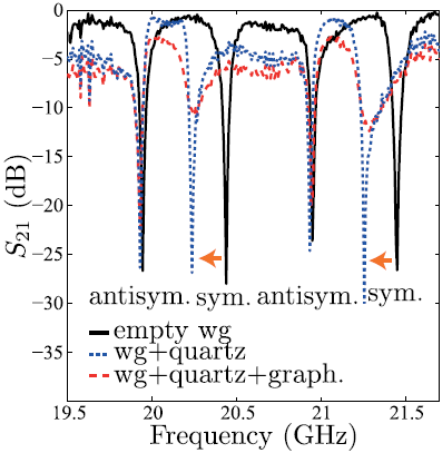
Isolators at GHz using graphene

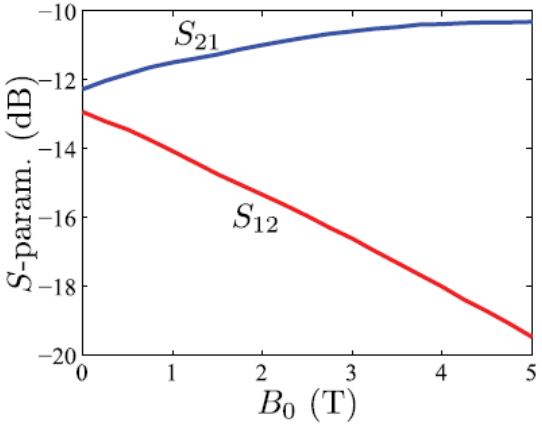












- Sounas, D. L. et al. Faraday rotation in magnetically biased graphene at microwave frequencies. APL (2013)
- Sounas, D. L. & Caloz, C. in IEEE International Symposium on Antennas and Propagation (APSURSI), 2011

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Outline

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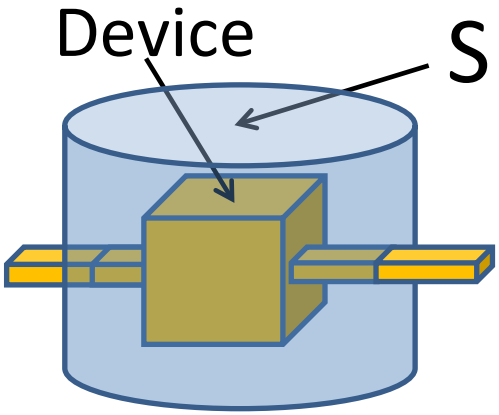
The theoretical bound



- Let us consider a 2 ports device:

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

- Let us define: $\boldsymbol{W} = \boldsymbol{E}_1 \times \boldsymbol{H}_2 - \boldsymbol{E}_2 \times \boldsymbol{H}_1$
- Divergence theorem:

$$\int_S W \cdot d\boldsymbol{S} = \int_V (\nabla \cdot W) dV$$

- We get

$$2\, \boldsymbol{a_B}^T (S - S^T) \boldsymbol{a_A} = \int_V (\nabla \cdot W) dV$$

M. Tamagnone, A. Fallahi, J. R. Mosig, and J. Perruisseau-Carrier, "Fundamental limits and near-optimal design of graphene modulators and non-reciprocal devices," Nature Photonics

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The theoretical bound

- Assuming the presence of a 2D non reciprocal material with conductivity:

$$\underline{\sigma} = \begin{pmatrix} \sigma_d & \sigma_o \\ -\sigma_o & \sigma_d \end{pmatrix}$$

- Assuming no other non reciprocal material
- Assuming linearity:

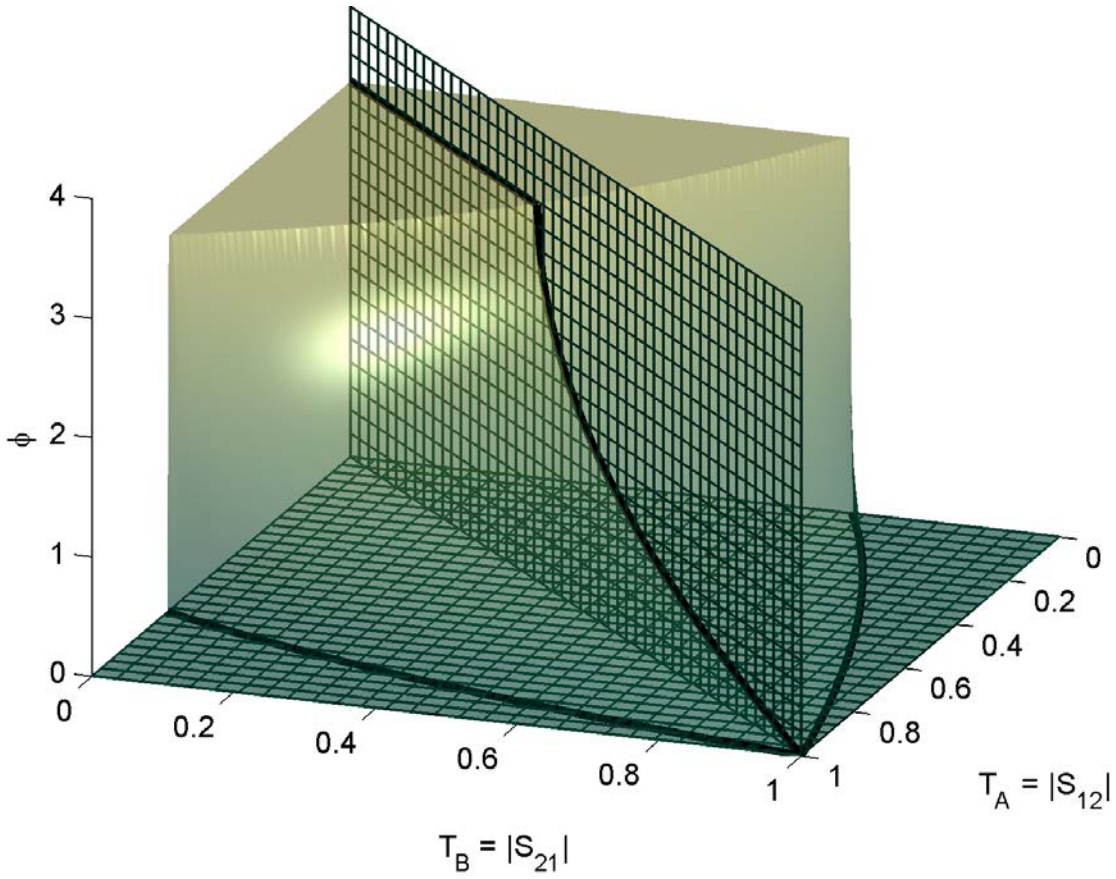
$$\gamma_{Dev} \leq \gamma_{NR}$$

$$\gamma_{Dev} = \frac{|S_{12} - S_{21}|^2}{(1 - |S_{12}|^2)(1 - |S_{21}|^2)}$$

$$\gamma_{NR} = \frac{|\sigma_o|^2}{Re^2(\sigma_d) - Im^2(\sigma_o)}$$

➢ M. Tamagnone, A. Fallahi, J. R. Mosig, and J. Perruisseau-Carrier, "Fundamental limits and near-optimal design of graphene modulators and non-reciprocal devices," Nature Photonics

The theoretical bound



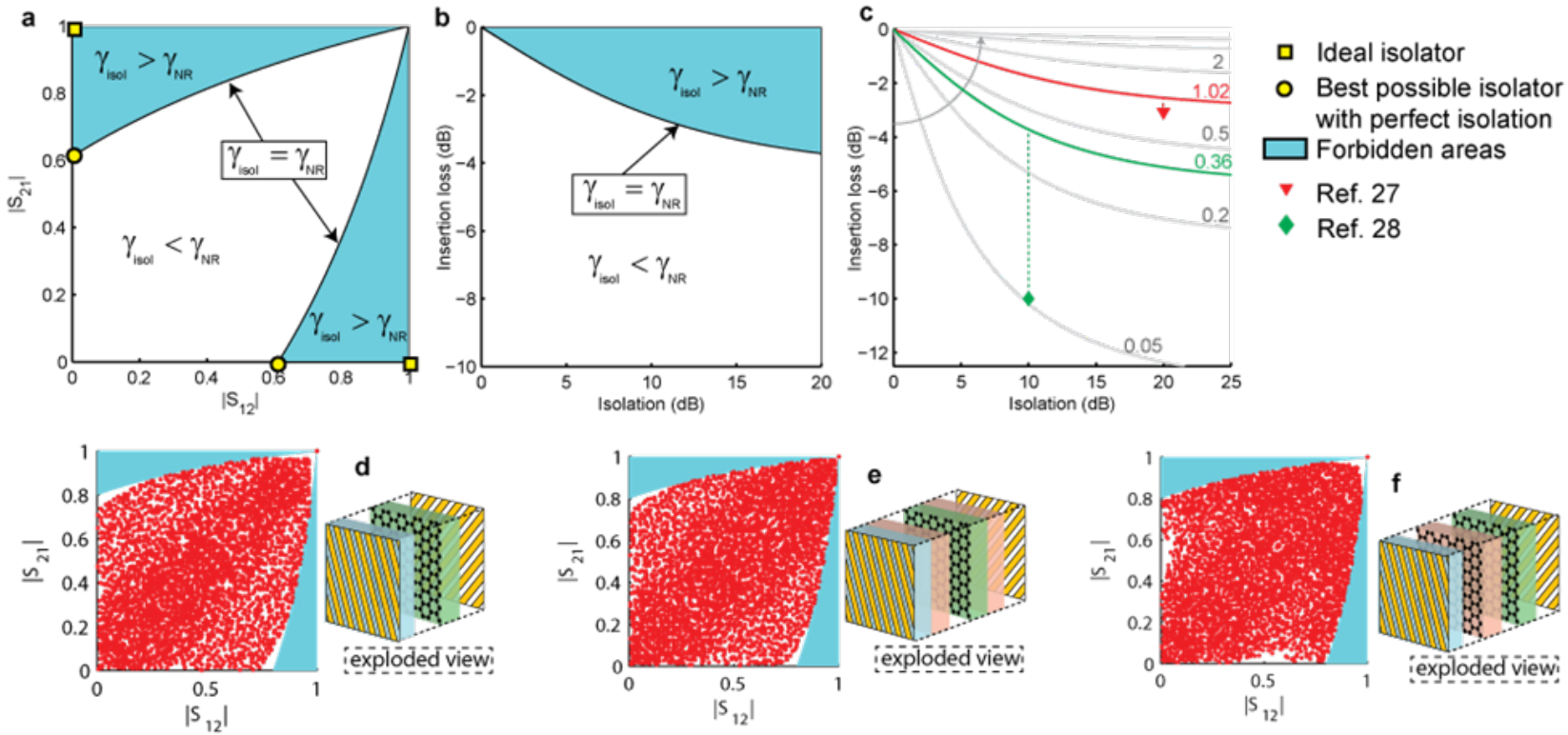
3 real coordinates:

- $T_A = |S_{12}|$
- $T_B = |S_{21}|$
- $\varphi = \angle S_{12} - \angle S_{21}$

$$\frac{|S_{12} - S_{21}|^2}{(1 - |S_{12}|^2)(1 - |S_{21}|^2)} = \frac{{T_A}^2 + {T_B}^2 - 2T_A T_B \cos\varphi}{\left(1 - {T_A}^2\right)\left(1 - {T_B}^2\right)} \leq \gamma_{NR}$$

➢ M. Tamagnone, A. Fallahi, J. R. Mosig, and J. Perruisseau-Carrier, "Fundamental limits and near-optimal design of graphene modulators and non-reciprocal devices," Nature Photonics

The theoretical bound



Random simulation data: 1 THz, $T = 3$ K, $\mu_c = 0.2$ eV, $B_0 = 4\ T$, and $\tau = 66\ fs$ ($\gamma_{NR} = 1.78$)

- M. Tamagnone, A. Fallahi, J. R. Mosig, and J. Perruisseau-Carrier, "Fundamental limits and near-optimal design of graphene modulators and non-reciprocal devices," Nature Photonics

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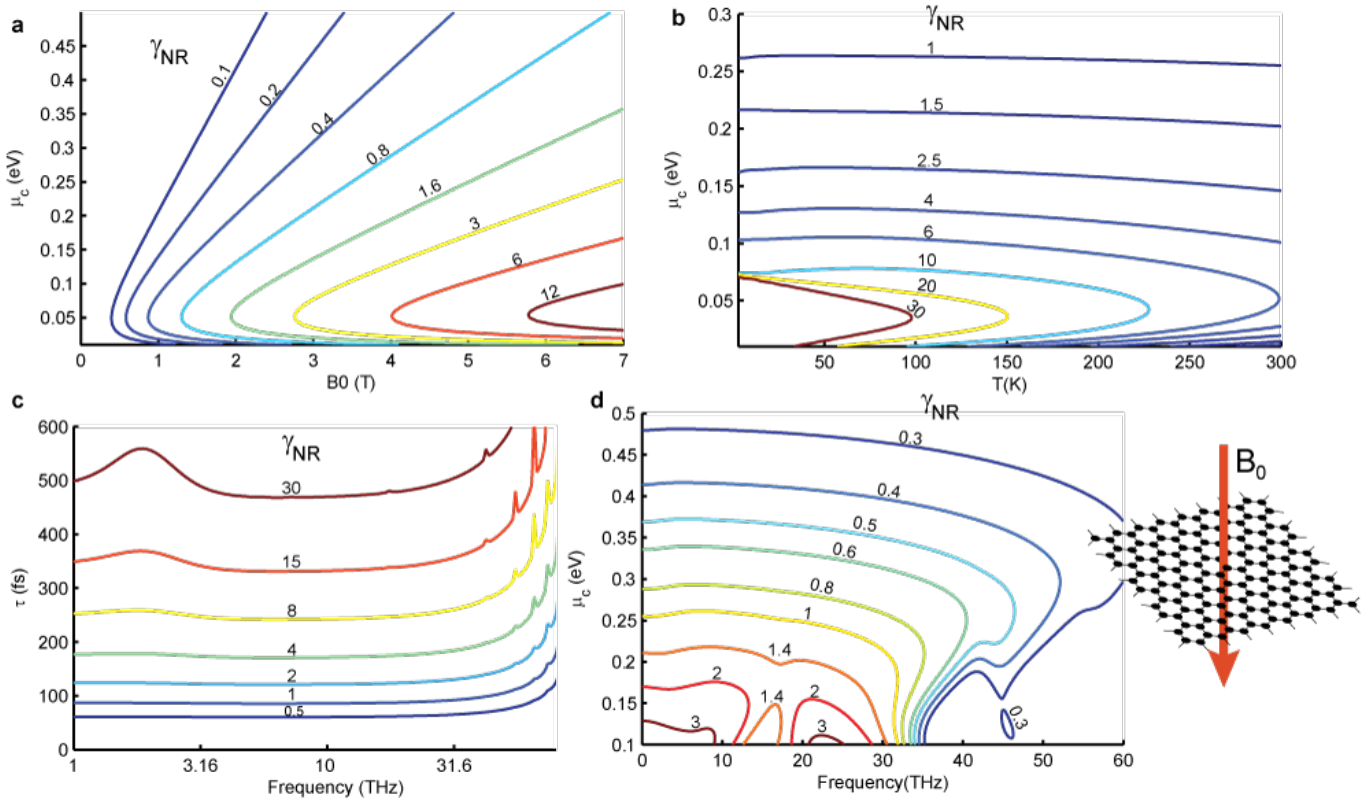
The theoretical bound



$$\gamma_{NR} = \frac{|\sigma_o|^2}{Re^2(\sigma_d) - Im^2(\sigma_o)} \cong (\omega_c \tau)^2 = (\mu B_0)^2 = \left(B_0 e \tau {v_f}^2\right)/\mu_c$$

Default: f=1 THz, T=300 K, μ_c=0.34 eV, B_0=4 T, and τ =66 fs

- M. Tamagnone, A. Fallahi, J. R. Mosig, and J. Perruisseau-Carrier, "Fundamental limits and near-optimal design of graphene modulators and non-reciprocal devices," Nature Photonics

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Extreme generality of the bound

- Limited number of requirements:
  - Linearity
  - Passivity
  - Reciprocity (for the reconfigurability case)
  - Scattering matrix representation (infinite ports allowed)
- The validity is independent of
  - Frequency
  - Device geometry (e.g. planar vs guided) and design complexity
  - The other materials in the device
- The presented bound is only a member of a wider family of bounds. (e.g. bound for tuneable graphene devices).

➢ M. Tamagnone, A. Fallahi, J. R. Mosig, and J. Perruisseau-Carrier, "Fundamental limits and near-optimal design of graphene modulators and non-reciprocal devices," Nature Photonics

DESY

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Extreme generality of the bound

The bound can be applied to a very large number of structures presented in literature:

- Graphene amplitude modulators (working at radiofrequencies, microwave, terahertz, far, mid and infrared)
- Non reciprocal devices (Isolator, Kerr and Faraday rotators, Faraday rotators based on patterned graphene)

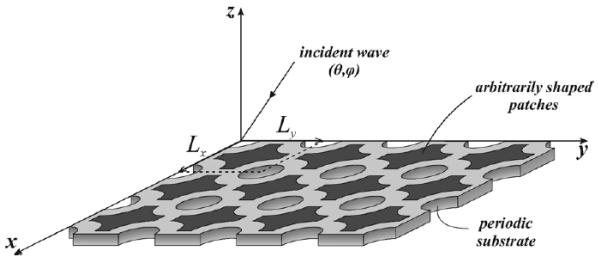
It can potentially be extended to:

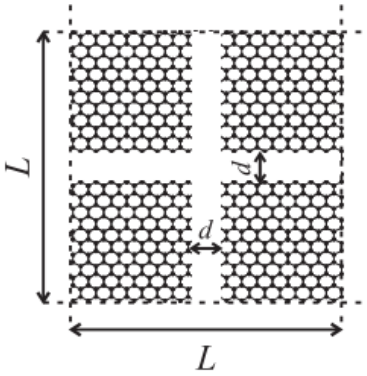
- Phase modulation
- Gyrators based on 2d material
- Reconfigurable graphene antennas.

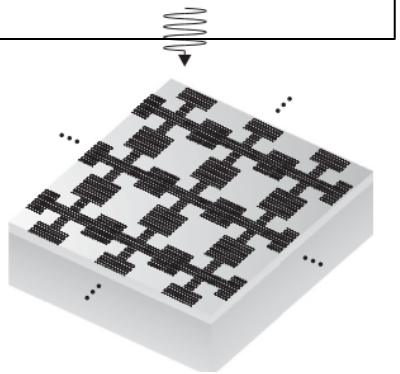
Random devices simulation and bound validation

- For planar structures a periodic MoM code has been used:

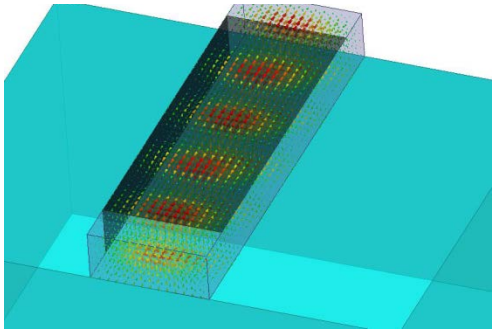
- A. Fallahi, et al. "Large overlapping subdomain method of moments for the analysis of frequency selective surfaces." Microwave Theory and Techniques, IEEE Transactions on 58.8 (2010): 2175-2187.
- A. Fallahi and J. Perruisseau-Carrier, "Design of tunable biperiodic graphene metasurfaces," *Physical Review B*, vol. 86, p. 195408, 2012.
- A. Fallahi and J. Perruisseau-Carrier, "Manipulation of giant Faraday rotation in graphene metasurfaces," *Applied Physics Letters*, vol. 101, pp. 231605-4, 2012.

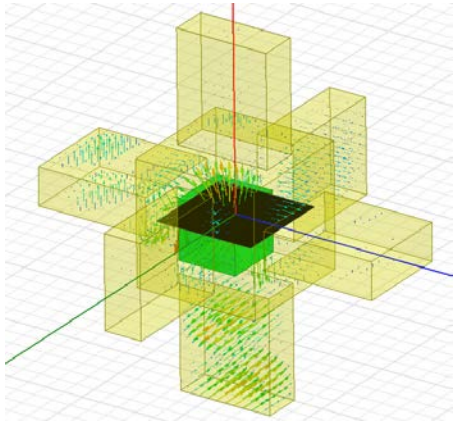






- HFSS validation of the bound for complex guided structures





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Conclusions

- There is a clear tradeoff between relevant figures of merit of non reciprocal devices. (e.g. insertion loss and isolation)
- Our upper bound identifies this tradeoffs mathematically
- It provides important physical insights
- It is extremely general
- It can be applied to other non-reciprocal materials

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DESY

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FÉDÉRALE DE LAUSANNE

In Memoriam
of Prof. Julien Perruisseau-Carrier
(1979-2014)

Thank you very much for attending.
Any questions?